Fast and Stable Explicit Operator Splitting Methods for Phase-Field Models

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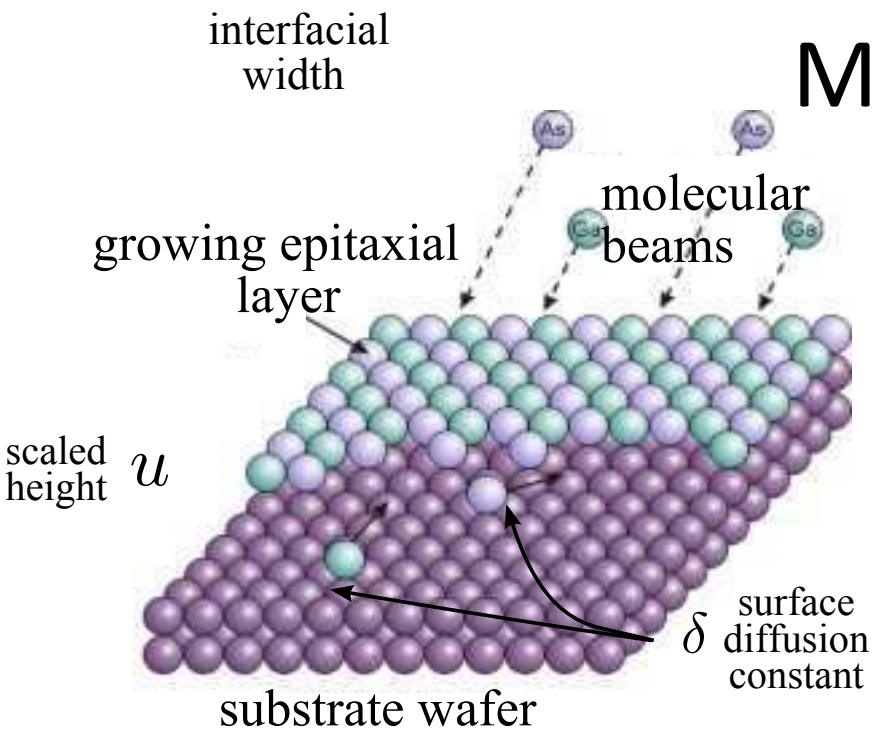
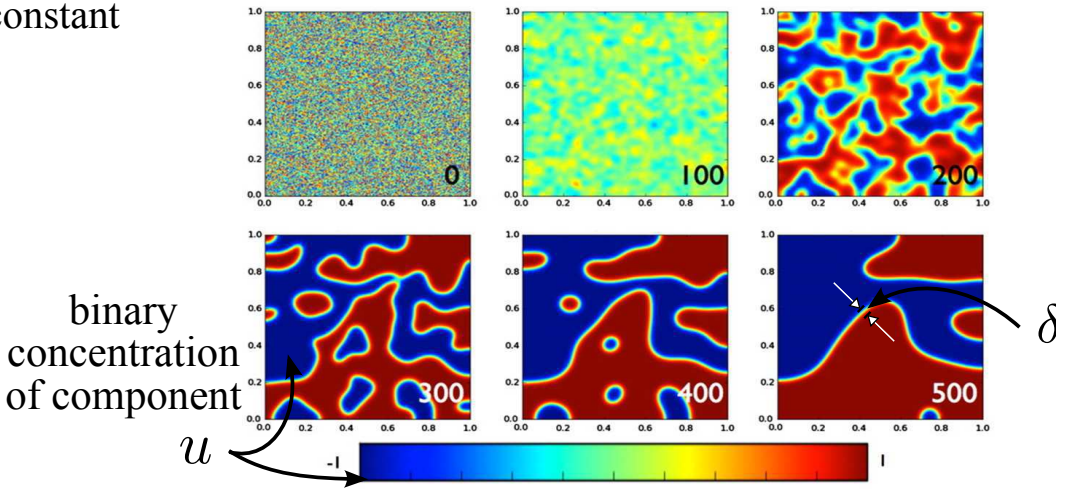


Molecular Beam Epitaxy (MBE)

Cahn-Hilliard (CH)



The equations can be viewed as the gradient flow of energy functionals

$$MBE: \quad u_t = -\delta\Delta^2 u - \nabla\cdot[(1-|\nabla u|^2)\nabla u]$$

$$E(u) = \int_\Omega \left[\frac{\delta}{2}|\Delta u|^2 + \frac{1}{4}(|\nabla u|^2-1)^2\right] d\boldsymbol{x}$$

$$CH: \quad u_t = -\delta\Delta^2 u + \Delta(u^3-u)$$

$$E(u) = \int_\Omega \left[\frac{\delta}{2}|\nabla u|^2 + \frac{1}{4}(u^2-1)^2\right] d\boldsymbol{x}$$

both energy functionals decay in time: $E(u(t)) \le E(u(s)),\ \forall t \ge s$

- Severe Timestep Restriction (accuracy): $\delta\Delta^2(\cdot)$
- Long-Time Simulations (efficiency): steady states
- Nonlinear Energy Stability: nonphysical oscillations