Fast and stable explicit operator splitting methods for phase-field models
Zhuolin Qu1, Yuanzhen Cheng3, Alexander Kurganov1 and Tao Tang2
1. Mathematics Department, Tulane University
2. Department of Mathematics, South University of Science and Technology of China

Develop fast and reliable numerical algorithms for phase-field models:
• MBE equation with step selection
  \[ u_t = -\Delta u + \nabla \cdot (V \nabla u); \quad x \in \Omega \subset \mathbb{R}^2 \]
  \[ \text{Cola-Bulle (CB) equation} \]
  \[ u_t = -\Delta u + \Delta u; \quad x \in \Omega \subset \mathbb{R}^2 \]
subs: periodic boundary conditions.

What are phase-field models?
Phase-field models are mathematical models to solve interfacial problems. The free energy: the deformation of a crystalline solid on a crystalline substrate is MBE equation (Figure 1).
Phase separation: two components of a binary fluid and independently segregate and form droplets in this component \( \rightarrow \) CB equation (Figure 1 right).

Numerical challenges
To numerically solve the models, we have following potential numerical difficulties:
• semi-discrete nonlinear part
  • linear part pseudospectral methods
  • nonlinear part semi-discrete finite-difference methods
  • numerical energy stability
  • adaptive splitting timestepping strategy

Efficient explicit and stable ODE solver DURKOB4 [7]:
Large time step accuracy.

Goal

To accurately solve dynamic, the perturbed inhomogeneous operator (\( \Omega \)), which is involved in the governing equation, may lead to severe instabilities on numerical timestep selection.

Adaptive splitting timestepping strategy
We adjust the size of splitting steps using the following roughness-dependent monitor function
\[ \Delta t = \max \left( \frac{\Delta t_{\text{min}}}{\alpha(x,t)}, \Delta t_{\text{max}} \right) \]
where \( \alpha = \text{constant}, x(t) = (1 - \frac{\| \nabla u \|^{2}}{\| \nabla u \|^2})^{\frac{1}{2}} \). (d)

Numerical examples

MBE equation \( (\theta = 0.5) \) subject to the initial condition \( u(x,0) = 0 \) for \( t = 5 \) and \( 0.5 \text{ sec} \), \( x \in [0,2] \).

• constant splitting timestep \( \Delta t = 10^{-3} \)
• adaptive splitting timestepping leads to similar results as above ones.

References

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