A hierarchy of reduced mathematical models for Wolbachia transmission in mosquitoes to control mosquito-borne diseases

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Abstract

We create and analyze reduced models for the spread of a Wolbachia bacteria infection in mosquitoes. Mosquitoes that are infected with Wolbachia are much less effective at transmitting zoonotic diseases. The infection persists in wild mosquitoes only if the fraction of infection exceeds a minimum threshold. Although the large system of differential equations capture the detailed transmission dynamics, they are difficult to analyze, which prevents practical extensions. The reduced models cut the analysis burden, yet capture the important properties of the original system. The parameters for the reduced models are defined in terms of the ones in full model.

Model Comparisons

Reduced models preserve two important properties of the detailed 9-ODE model.

Basic reproductive number $R_0$

- number of new infections created by a single Wolbachia-infected mosquito, given all the rest mosquitoes are fully susceptible
- often used as a threshold condition for disease outbreak (predict early epidemic)

$\begin{align*}
R_0^{(9)} &= v_w \frac{\sigma + \mu_u}{\mu_w} \\
R_0^{(7)} &= R_0^{(4)} = R_0^{(2)}
\end{align*}$

Critical threshold for a stable Wolbachia infection

- a minimum threshold must be exceeded to establish a stable infection in wild mosquitoes
- backward bifurcation with an unstable steady-state

We need to create a stable infection in wild mosquitoes:

- field trials in Australia, Brazil, Colombia, Indonesia, Vietnam to control dengue/Zika epidemics
- Fraction of infection has to exceed a critical threshold.

Maternal transmission Wolbachia is maternally transmitted from infected mothers to offspring.

- Wolbachia A promising strategy to stop diseases at source.
  - a natural parasitic microbe, found in 60% insects species
  - inhibits the proliferation of viruses inside the mosquito ⇒ blocks the disease transmission
- is not found in the wild Aedes aegypti mosquitoes

We need to create a stable infection in wild mosquitoes:

- nearly 700 million people get a mosquito-borne disease each year resulting in greater than one million deaths
  - Aedes aegypti mosquito: the primary vector for dengue fever, chikungunya and Zika

Mosquito-borne Diseases v.s. Wolbachia

- Aedes aegypti mosquitoes
  - cyttoplasmic incompatibility (CI)
  - $\bullet$ uninfected mosquitoes
  - $\bullet$ Wolbachia-infected mosquitoes
  - female  $\sigma$ male
  - $\bullet$ maternal transmission Wolbachia
  - $\bullet$ uninfected mosquitoes
  - $\bullet$ Wolbachia-infected mosquitoes
  - female  $\sigma$ male
  - $\bullet$ cyttoplasmic incompatibility (CI)

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